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A MULTITYPE CONCENTRATOR LOCATION PROBLEM WITH A CHOICE OF TERMINAL COVERAGES

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ABSTRACT

In recent years we have witnessed remarkable progress in the development of the topological design of computer communication networks. One of the important aspects of the topological design of computer communication networks is the concentrator location problem. When system reliability and availability are critical in computer communication networks, it may be desirable to assign additional concentrators as backups to a terminal. The extended version of the multitype concentrator location problem is considered in which various terminal coverages are allowed to improve reliability and availability with a choice of concentrator type. This problem is a complex combinatorial problem that belongs to the difficult class of NP-complete problems where the computation of an optimal solution is still a challenging task. A decomposition-based algorithm is proposed. Computational results are quite satisfactory and encouraging and show this algorithm to be both efficient and effective.

INTRODUCTION

The concentrator Location problems in computer communication networks have attracted a substantial amount of research attention during the last decade [1,2, 6,14,26,28,31,33,36,39,40]. A typical system is often characterized by a three-level hierarchy - terminals, "concentrators", and one CPU or central site. It is assumed that the terminal locations are known and that the CPU location is also known. It is also assumed that the cost of connecting a terminal to a concentrator, and the cost of concentrators are known. The concentrator location problem is the problem of determining the number and location of concentrators and allocating terminals among these concentrators without violating capacities of concentrators while at the same time keeping costs at a minimum. These concentrators can represent "office control units", which serve to provide common buffering and message handling for terminals at a site.

In this paper, the extended version of the classical concentrator location problem is considered in which various terminal coverages are allowed to improve reliability and availability with a choice of concentrator type. This is called a multitype concentrator location problem with a choice of terminal coverages. The analysis will be confined to a two-stage system (i.e., three-level hierarchy) - one central site, "concentrators", and terminals. This problem involves a network system with a choice of various types of concentrators and a choice of various terminal coverages. This model allows one to select a specific type of concentrator from among several standardized types. Each concentrator type offers a different capacity with different fixed set-up costs. Each terminal has one or more concentrators assigned to it. In the classical version of the problem, where each terminal is assigned to a single concentrator site, a link (line) is set up from each terminal to each concentrator, and the latter in turn is connected to the central site. Under this scheme a terminal can be cut off from the network if one or more of the following events occur: the link from a terminal to its concentrator fails, the concentrator itself fails, or the link from the concentrator to the central site fails. When system reliability and availability are crucial in computer communication networks , it may be desirable to assign additional concentrators to a terminal. In such a network each terminal i will be assigned to L_i ($L_i \geq 1$ and integer) concentrators, creating L_i disjoint paths to the central site and thereby substantially decreasing the probability that a terminal is not connected to the central site. The choice of the value for L_i for terminal i depend on how critical it is that terminal i be connected to the central site. Typically a higher value of L_i would indicate that the terminal is deemed more critical than a terminal with a lower value.

LITERATURE REVIEW

The concentrator location problem is a well-known facility location problem of both theoretical and practical interest [1,2,4,6,7,10,12-14,17,18,26-29,31,33, 36,39,40]. Virtually all of these models are (mixed) integer linear programming problems. The result is a complex combinatorial model which belongs to the class of NP-complete problems [11]. Most of the algorithms developed in the past have been based on either branch-and-bound techniques with or without Lagrangean relaxation [12,17,28,29,31,39] or heuristic approaches [2,6,27,33,36,40].

Recently, several applications of Benders decomposition to computer communication network problems have been proposed [12,15,23]. Benders decomposition methods, or primal decomposition methods, exploit only the primal structure of the problem. However, it has been observed that many (mixed) integer programming problems have both easy-to-solve primal and dual subproblems. This motivates the development of an algorithm based on cross decomposition. Cross decomposition allows one to exploit both structures of the primal and dual subproblems simultaneously.

The cross decomposition scheme has been successfully applied to many combinatorial optimization problems during the last decade. The use of cross decomposition was first suggested by Van Roy [37]. The successful application of this decomposition to the facility location problem by Van Roy [38] led to its use in a number of problems. Various location problems [16, 20,21,22,24,38,41], a vehicle routing problem [32], and a distributed computer system design problem [23] are only a few of the problems to which cross decomposition has been applied.

MODEL DESCRIPTION

The multitype concentrator location problem with various terminal coverages is a concentrator location problem with a choice of concentrator types and terminal coverages. The number of terminals and their locations are assumed to be known. User demand requirements and terminal coverages are also assumed to be given. The potential concentrator locations and the capacity of each concentrator are also known. Each terminal can be connected to any one of the concentrators for its primary coverage, to any other concentrator for its secondary coverage, and so on. The problem is to determine for every potential concentrator location, whether to open a concentrator or not, and in case a concentrator is open, which terminals will have it as their primary concentrator, which ones will have it as their secondary concentrator, and so on, without violating capacities of concentrators at a minimum cost.

The following notation will be used in the model:

- I = index set of terminal locations; $i \in I$,
- J = index set of potential concentrator locations; $j \in J$,
- K = index set of standardized concentrator types; $k \in K$,
- L_i = index set of the number of concentrators to be assigned to terminal i; $l \in L_i$,
- c_{ijl} = cost of assigning terminal i to the concentrator at j as the lth concentrator,
- m_j = cost of operating a unit of concentrator capacity at site j,
- a_{ijl} = amount of concentrator capacity required to support terminal i if the concentrator at site j is designed as the lth concentrator for that terminal i,
- f_{jk} = fixed cost of locating a concentrator type k at site j,
- b_{jk} = maximum capacity of the concentrator type k at site j,

$$X_{ijl} = \begin{cases} 1 & \text{if } j \text{ is the } l\text{th concentrator site for terminal } i, \\ 0 & \text{otherwise,} \end{cases}$$

$$Y_{jk} = \begin{cases} 1 & \text{if a concentrator type } k \text{ is located at site } j, \\ 0 & \text{otherwise,} \end{cases}$$

The problem can be described more specifically as follows: Given a set of terminals $i \in I$, select a subset of possible concentrator sites $j \in J$, where the capacity of a concentrator type k at node j, b_{jk}, can be chosen from among $\{b_{jk}: k \in K\}$, and assign terminals to these concentrators with various terminal coverages allowed so as to minimize the cost of locating concentrators and assigning terminals. Terminals can also be assigned directly to the central site, which can be considered as a distinguished concentrator. Each terminal is assigned to at least one concentrators depending on the required coverage. The mathematical formulation is presented as follows:

$$\text{Min} \quad \sum_i \sum_j \sum_l c_{ijl} X_{ijl} + \sum_i \sum_j \sum_l m_j a_{ijl} X_{ijl} + \sum_j \sum_k f_{jk} Y_{jk} \tag{1}$$

$$\text{S.T.} \quad \sum_j X_{ijl} = 1 \qquad \forall i \in I, \forall l \in L_i \tag{2}$$

$$\sum_{k} Y_{jk} \leq 1 \qquad \forall j \in J \tag{3}$$

$$\sum_{i}\sum_{l} a_{ijl}X_{ijl} \leq \sum_{k} b_{jk}Y_{jk} \qquad \forall j \in J \tag{4}$$

Linear configuration constraints on Y's (5)

$$X_{ijl} = 0,1 \qquad \forall i \in I,\ \forall j \in J,\ \forall l \in L_i \tag{6}$$

$$Y_{jk} = 0,1 \qquad \forall j \in J,\ \forall k \in K \tag{7}$$

The first component of the objective function represents the cost of the assignments of terminals to concentrators. The second component captures operating costs that are directly related to the size of the concentrator required to serve terminals, and the third component is a fixed set-up cost of establishing concentrators; it is assumed to be independent of the concentrator size. For the sake of generality, the above model presents concentrator requirements for each terminal as a_{ijl} implying that the capacity consumption for the *l*th assignment for terminal *i* may be dependent on the concentrator the terminal is assigned to. The a_{ijl} parameters which express the capacity demand for the various "backup" concentrators could be estimated as a function of the probability of failure of the links from terminal *i* to concentrators, the probability of failure of the concentrators themselves, the probability of failure of the link from the concentrator to the central site, and also of the durations of these failures.

Constraint set (2) ensures that each terminal is assigned L_i concentrators. Constraint set (3) assures that at most one type be selected for each concentrator site j. Constraint set (4) ensures that the total capacity required by all terminals assigned to each concentrator does not exceed its capacity. Linear configuration constraint set (5) can be used to impose a variety of requirements. Constraint sets (6) and (7) enforces the integrality conditions on the decision variables.

For convenience, the linear configuration constraint set (5) is ignored from now on. Consequently, the model can be rewritten as the following problem P:

P

$$\min_{X,Y=0,1} \sum_{i}\sum_{j}\sum_{l} c_{ijl}X_{ijl} + \sum_{i}\sum_{j}\sum_{l} m_j a_{ijl}X_{ijl} + \sum_{j}\sum_{k} f_{jk}Y_{jk} \tag{8}$$

$$\text{S.T.} \quad \sum_{j} X_{ijl} = 1 \qquad \forall i \in I,\ \forall l \in Li \tag{9}$$

$$\sum_{k} Y_{jk} \leq 1 \qquad \forall j \in J \tag{10}$$

$$\sum_{i}\sum_{l} a_{ijl}X_{ijl} - \sum_{k} b_{jk}Y_{jk} \leq 0 \qquad \forall j \in J \tag{11}$$

DECOMPOSITION OF THE PROBLEM

As mentioned earlier the model presented in the previous section belongs to the class of NP-complete problems [11]. Development of an effective solution algorithm for this model depends on being able to exploit its special structure.

Resource-directive decomposition (Primal or Benders decomposition)

The primal subproblem, denoted by SP_Y, is obtained by fixing in the problem P the variables Y_{jk} to either 0 or 1:

SP_Y

$$\min_{X=0,1} \sum_{i}\sum_{j}\sum_{l} (c_{ijl} + m_j a_{ijl})X_{ijl} \tag{12}$$

$$\text{S.T.} \quad \sum_{j} X_{ijl} = 1 \qquad \forall i \in I,\ \forall l \in L_i \tag{13}$$

$$\sum_{i}\sum_{l} a_{ijl}X_{ijl} \leq \sum_{k} b_{jk}Y_{jk} \qquad \forall j \in J \tag{14}$$

For this problem several efficient solution methods exist [3,5,35]. To determine the values Y_{jk} to be used in SP_Y, the Benders master problem MP_μ must be solved:

MP_μ

$$\min_{Y=0,1} W \tag{15}$$

$$\text{S.T.} \quad W - \sum_{j}\sum_{k} (f_{jk} - b_{jk}\mu_j^t)Y_{jk} \geq \sum_{i}\sum_{l} \nu_{il}^t \qquad \forall t \in T_{PA} \tag{16}$$

$$\sum_{j} Y_{jk} \leq 1 \qquad \forall j \in J \tag{17}$$

where T_{PA} is the index set of all dual feasible basic solutions (ν_{il}^t, μ_j^t) of SP_Y and ν_{il} and μ_j correspond to the constraint sets (9) and (11). The constraints indexed by $t \in T_{PA}$ are called Benders or primal cuts.

Price-directive decomposition (Dual decomposition or Lagrangean relaxation)

The generalized upper bound constraint set (9) and the concentrator location constraint set (10) represent the dual structure which can be exploited, for instance, via a Lagrangean relaxation [9] with respect to the capacity constraint set (11). In other words, the dual subproblem, denoted SD_μ, is obtained by fixing in the problem P the dual variable μ_j corresponding to the constraint set (11):

SD_μ

$$\min_{X,Y=0,1} \sum_{i}\sum_{j}\sum_{l} (c_{ijl} + m_j a_{ijl} + \mu_j a_{ijl})X_{ijl} + \sum_{j}\sum_{k} (f_{jk} - \mu_j b_{jk})Y_{jk} \tag{18}$$

$$\text{S.T.} \quad \sum_{j} X_{ijl} = 1 \qquad \forall i \in I,\ \forall l \in L_i \tag{19}$$

$$X_{ijl} \leq \sum_{k} Y_{jk} \qquad \forall i \in I,\ \forall j \in J,\ \forall l \in L_i \tag{20}$$

$$\sum_{k} Y_{jk} \leq 1 \qquad \forall j \in J \tag{21}$$

$$\sum_{j}\sum_{k} b_{jk}Y_{jk} \geq \sum_{i}\sum_{j}\sum_{l} a_{ijl} \tag{22}$$

Note that the generalized upper bound constraint set (20), which requires that no terminals be assigned to the closed nodes (i.e. nodes without concentrators), is added. This constraint set (20) is redundant to the problem P, but this additional constraint set leads to a tighter formulation which results in a more successful application of the Lagrangean relaxation. Also, the surrogate constraint set (22) is included in the dual subproblem, SD_μ. This surrogate constraint set (22) enforces the solution of the dual subproblem, SD_μ, to meet the feasibility condition that the total capacity required by all terminals should not exceed the total capacity of concentrators in the system.

The dual subproblem, SD_μ, takes the form of an ordinary uncapacitated facility location problem if the surrogate constraint set (22) is not included. Erlenkotter's dual-based solution method [8] is known to be a most powerful technique. Now associating the dual variables ν_{il}, ω_{ijl}, and π with the constraint sets (19), (20), and (22) respectively, and by letting

$$\omega_{ijl} = \max \{ 0,\ \nu_{il} - (c_{ij} + m_j a_{ijl} + \mu_j a_{ijl}) \}$$

the following condensed dual of the linear programming relaxation of SD_μ, DSD_μ, can be obtained:

DSD_μ^1

$$\max \sum_{i}\sum_{l} \nu_{il} + \Big(\sum_{i}\sum_{j}\sum_{l} a_{ijl}\Big)\pi \tag{23}$$

$$\text{S.T.} \quad \sum_{i}\sum_{l} \omega_{ijl} \leq \bar{f}_{jk} - b_{jk}\pi \qquad \forall j \in J,\ \forall k \in K \tag{24}$$

$$\nu_{il} \text{ unrestricted in sign} \qquad \forall i \in I,\ \forall l \in L_i \tag{25}$$

$$\pi \geq 0 \tag{26}$$

where $\bar{f}_{jk} = f_{jk} - \mu_j b_{jk}$ for $\forall j \in$ and $\forall k \in K$. Once π is given, the above dual problem, DSD_μ, can be reduced to a condensed dual of a relaxed linear program of an ordinary uncapacitated facility location problem for which the dual-based solution method is directly applicable. Therefore, the solution method consists of two phases: the first phase determines the value of π which satisfies the surrogate constraint set (22), and the second phase applies the dual-based method with the given π. The first phase is similar to a binary-type search of Van Roy [38], which is based on the property, due to the constraint set (24), that the number of concentrators increases as π increases. Once π is obtained from the first phase, the size of the resulting uncapacitated facility location problem is substantially reduced to a facility location problem with the fixed facility capacity by the following transformation: If $\min_k \{\bar{f}_{jk}\} < 0$ for some j, then Y_{jh} is fixed to 1 where $h = \arg\min_k \{\bar{f}_{jk}\}$. Otherwise, set $\hat{f}_j = \min \{f_{jk} - b_{jk}\pi\}$ for all j and eliminate row $k \in \{ K \setminus k' \}$ for each j from the constraint set (24) where $k' = \arg\min_k \{\bar{f}_{jk} - b_{jk}\pi\}$ for each j.

The price μ_j can be determined by the solution of a dual master problem, or Lagrangean master problem, MD_Y:

MD_Y

$$\max_{\mu \geq 0} \delta \tag{27}$$

$$\text{S.T.} \quad \delta - \sum_{j} \Big(\sum_{i}\sum_{l} a_{ijl}X_{ijl}^t - \sum_{k} b_{jk}Y_{jk}^t\Big)\mu_j \leq \sum_{i}\sum_{j}\sum_{l} (c_{ijl} + m_j a_{ijl})X_{ijl}^t + \sum_{j}\sum_{k} f_{jk}Y_{jk}^t$$

$$\forall t \in T_{DA} \tag{28}$$

where T_{DA} is the index set of feasible solutions (X^t_{ijl}, Y^t_{jk}) of SD_μ. The constraints $t \in T_{DA}$ are called dual cuts.

CROSS DECOMPOSITION

In the previous section two subproblems with special structures were derived. Price-directive decomposition (Lagrangean or Dantzig-Wolfe decomposition) creates a subproblem that takes advantage of the special structure of the ordinary uncapacitated facility location problem. Resource-directive decomposition (Benders decomposition) exploits another special structure of the generalized assignment problem. The difficult parts of both cases are the solutions to the master problems. However, as Van Roy [37] shows, the primal subproblem SP_Y and the dual subproblem SD_μ are relaxed master problems for each other. Although the subproblems $SP_{Y,Z}$ and SD_μ are master problems for each other, it does not follow that the problem P can be solved just by alternating between both subproblems. For example, the algorithm would never be able to terminate if the Lagrangean relaxation has a nonzero duality gap, and as in the simplex method cycling may occur if no precautions are taken (even if the Lagrangean relaxation has no duality gap). Therefore convergence tests are needed. Convergence tests in the proposed solution algorithm below prevent the algorithm from cycling, and from solving the primal master problem at each iteration.

Solution Procedure

A cross decomposition-based algorithm for a multitype concentrator location problem with a choice of various terminal coverages is now given in summary form:

Step 1) Initialization. Get initial values of the dual variables, $\mu^{(0)}$.
Set $k = 0$.

Step 2) Set $k = k+1$, solve the dual subproblem, SD_μ, with the given dual variables, $\mu^{(k-1)}$. The primal solution achieved is $X^{(k)}$ and $Y^{(k)}$, and the optimal objective function value will give a lower bound on the objective function value of problem P. Perform an optimality test: if the upper and the lower bounds on the objective function are close enough, then stop: the latest saved solution is close enough to the optimum to be regarded as optimal; if not, continue.

Step 3) Perform convergence test 1: test whether $Y^{(k)}$ can lead to a better solution in the Benders subproblem. That is, if

$$\sum_j\sum_k (f_{jk} - b_{jk}\mu^t_j)Y_{jk} + \sum_i\sum_l \nu^t_{il} \geq V_P$$

where V_P denotes the current best (or incumbent) upper bound, then go to step 4; if not, go to step 6.

Step 4) Solve Benders subproblem, $SP_{Y,Z}$, with the given primal solution, $Y^{(k)}$. The primal solution is a feasible solution, and will be saved as a candidate for the optimum of the master problem. The objective function value is consequently an upper bound for the final optimal objective function value. The dual solution achieved is $\mu^{(k)}$. Perform the optimality test as above.

Step 5) Perform convergence test 2: check whether $\mu^{(k)}$ can lead to a better solution in the dual subproblem. That is, if

$$\sum_j \Big(\sum_i\sum_l a_{ijl}X^t_{ijl} - \sum_k b_{jk}Y^t_{jk}\Big)\mu_j + \sum_i\sum_j\sum_l (c_{ijl} + m_j a_{ijl})X^t_{ijl} + \sum_j\sum_k f_{jk}Y^t_{jk} \leq V_D$$

where V_D denotes the current best (or incumbent) lower bound, then go to step 2; if not, go to step 6.

Step 6) Solve one of the master problems. If convergence test 1 fails, the primal master problem, MP_μ, is solved. It will give a primal solution, $Y^{(k)}$, that satisfies convergence test 1, and also a lower bound on the objective function value. Perform the optimality test. Go to step 4. If convergence test 2 fails, the dual master problem MD_Y is solved. It will give a dual solution, $\mu^{(k)}$, that satisfies convergence test 2, and also an upper bound on the objective function value. The primal solution is primal feasible and is saved. Perform the optimality test. Go to step 2.

COMPUTATIONAL RESULTS

The proposed algorithm was coded in FORTRAN-77 and experiments were performed using an IBM 3090 Model 200 E. The main constituents of the program are the following four subroutines: (1) an efficient procedure for solving the generalized assignment subproblem [35], (2) a dual-based procedure for the Lagrangean subproblem, which is obtained by slightly modifying that of Van Roy [38], (3) a revised simplex method for the dual master problem, and (4) an implicit enumeration method for the primal master problem.

In order to evaluate the performance of the algorithm, a set of computational experiments were performed. The data used in these experiments were generated to conform to the primary, secondary, tertiary, etc. The parameters L_i were randomly selected with equal probability for all the integers between 1 and either 2, 3, or 5. The primary capacity requirement a_{ij1} was drawn from a uniform distribution between 10 and 20. The other capacity requirements a_{ij2}, a_{ij3}, a_{ij4}, and a_{ij5}, if needed, were fixed at 0.30, 0.20, 0.10, and 0.05 times the value of a_{ij1}, respectively. Fixed cost, f_{jk}, were randomly generated via a uniform distribution on the interval [1000, 10000], and variable costs, both c_{ijl} and m_j, were also randomly generated via a uniform distribution on the interval [50, 500], and [5, 10], respectively. The maximum capacity of concentrator, b_{jk}, was set by 100*(k+1) for the type k.

Table 1 displays summarized information for a set of test problems by comparing the CPU execution times, in seconds, needed to solve problems optimally by the proposed cross decomposition algorithm (CD), by the embedded Benders decomposition algorithm (BEN) [20], and by the branch-and-bound algorithm (B&B) [34] on an IBM 3090 Model 200E. Unfortunately, no published computational results for the problem P are available for purposes of comparison. However, it can be concluded that the performance of the proposed algorithm is quite satisfactory and encouraging considering the CPU times with the embedded Benders decomposition algorithm. Benders decomposition algorithm is generally known as one of the most efficient and powerful method for the class of large-scale (mixed) integer programming problems in facility location problem [19,20,25,30].

Table 1
Summarized Comparisons of Optimal CPU times

No	$\|I\|/\|J\|/\|K\|/\|L_i\|$				CD	BEN	B&B
1	20	5	2	1	1.24	1.29	6.38
2	20	5	2	2	1.69	2.42	8.06
3	20	5	3	2	2.37	2.92	11.53
4	20	5	3	3	1.92	2.90	12.11
5	30	5	2	1	2.43	2.63	11.51
6	30	5	2	2	2.68	3.09	15.17
7	30	5	3	2	2.61	3.42	20.12
8	30	5	3	3	2.95	3.74	22.88
9	40	5	2	1	3.44	5.09	27.25
10	40	5	2	2	3.27	4.44	25.86
11	40	5	3	2	3.84	5.96	30.05
12	40	5	3	3	3.62	5.21	26.39
13	40	10	3	2	5.39	8.44	42.63
14	40	10	3	3	6.75	10.19	53.52
15	40	10	5	3	6.81	11.33	51.61
16	40	10	5	5	7.93	12.59	64.47
17	50	5	2	1	6.61	10.95	57.13
18	50	5	2	2	7.37	12.46	65.72
19	50	5	3	2	8.24	15.87	63.15
20	50	5	3	3	7.68	15.31	61.70
21	50	10	3	2	9.13	15.86	77.81
22	50	10	3	3	8.87	17.19	70.54
23	50	10	5	3	8.49	14.93	82.32
24	50	10	5	5	9.77	16.60	89.71
25	50	20	3	2	12.57	24.31	115.53
26	50	20	3	3	13.84	21.48	121.47
27	50	20	5	3	13.05	25.57	112.63
28	50	20	5	5	13.68	25.12	129.24
29	75	10	3	2	22.96	45.77	183.86
30	75	10	3	3	24.18	40.35	>>
31	75	10	5	3	28.41	48.61	>>
32	75	10	5	5	27.19	50.87	>>
33	75	20	3	2	27.83	43.45	>>
34	75	20	3	3	30.66	52.57	>>
35	75	20	5	3	33.79	54.18	>>
36	75	20	5	5	37.25	58.47	>>
37	100	10	3	1	56.37	98.16	>>
38	100	10	3	2	59.12	115.85	>>
39	100	10	5	1	54.36	97.67	>>
40	100	10	5	2	62.49	123.30	>>

>> indicates CPU time exceeds 200 seconds.

No = problem number.

$|I|/|J|/|K|/|L_i|$ = number of terminals/number of potential concentrator locations/number of concentrator types/number of terminal coverage at terminal i.

CD = CPU time needed by the proposed cross decomposition algorithm (in seconds).

BEN = CPU time needed by the embedded Benders algorithm (in seconds).

B&B = CPU time needed by the branch-and-bound algorithm (in seconds).

The CPU time needed to solve the problems of this size by the proposed algorithm is less than 63 seconds. Thus, the algorithm solves problems of practical sizes in a reasonable amount of time. From Table 1, it is apparent that the CPU time using the proposed algorithm is substantially less than that by the embedded Benders decomposition algorithm and by the branch-and-bound algorithm. It can be concluded that the performance of the proposed algorithm is quite satisfactory and encouraging considering the comparison of CPU times with the above related alternatives although no other published computational results for problem P are available.

For all the problems solved, tight lower and upper bounds were obtained with a small gap in just a few iterations of the proposed algorithm. Consequently, for large size problems, a truncated algorithm can be used as a heuristic that produces not only a feasible solution, but also a confidence interval to measure the quality of the solution. Table 2 supports the truncated algorithm as an effective heuristic algorithm. In Table 2 the results for problems with up to 200 terminals and 30 potential concentrator locations are displayed. A total of 320 problems were solved (10 problems for each dimension). They were arranged into groups of 10 where all problems in a group were generated with the same structure in order to achieve a reasonable level of confidence about the performance of the procedure on that problem structure. When the gap between upper and lower bounds obtained from both subproblems remains persistent after a certain number of iterations, the proposed algorithm is truncated. This truncated algorithm is called the CD heuristic. For each group of problems the minimum, mean, and maximum gap values are reported. The average CPU times are also reported. The accuracy measure of percentage gap between the best primal and the best dual bound obtained when the algorithm is truncated, i.e., [(UB - LB)/UB] X 100 % , is also given to represent the effectiveness of the heuristic. The gap between the best primal feasible solution value (upper bound) and the best dual feasible solution value (lower bound) is used to judge the quality of the algorithm. The average number of concentrators opened are also reported. These results indicate that the truncated algorithm is effective in solving large size problems, with mean gaps less than 3% which is reasonably acceptable.

Table 2

Summarized Performance of the Proposed Algorithm

Problem Size				Percentage gap between the upper and the lower bound			Mean CPU time (in seconds)	Average no. of concentrators opened
$\|I\|$	$\|J\|$	$\|K\|$	$\|L_i\|$	Min.	Mean	Max.		
100	10	3	3	0.17	1.38	2.53	31.49	6.2
			5	0.12	1.87	3.62	39.74	7.8
		5	3	0.29	1.79	3.66	56.38	8.3
			5	0.32	1.95	4.10	63.27	8.4
100	20	3	3	0.38	1.48	2.63	55.35	11.8
			5	0.22	0.87	2.73	68.62	14.7
		5	3	0.39	0.99	2.66	79.83	16.2
			5	0.44	2.17	3.55	81.78	18.1
150	10	3	3	0.13	0.76	1.29	54.31	5.3
			5	0.04	1.22	3.47	60.74	6.1
		5	3	0.72	2.09	4.32	69.87	6.8
			5	0.53	1.42	3.75	75.26	7.5
150	20	3	3	1.23	1.64	2.76	73.49	11.0
			5	1.45	2.65	3.73	81.56	13.2
		5	3	0.53	1.39	3.81	88.32	13.9
			5	0.95	1.14	2.44	101.48	16.6
150	30	3	3	0.86	2.70	5.87	128.53	17.2
			5	0.69	1.99	4.36	159.66	22.1
		5	3	0.77	2.29	4.78	188.71	20.4
			5	0.84	2.58	2.83	204.24	24.8
200	10	3	3	0.64	1.80	3.31	98.76	5.2
			5	0.71	2.20	3.55	114.37	5.9
		5	3	0.75	1.89	3.09	129.58	6.5
			5	0.83	3.03	4.49	143.76	7.4
200	20	3	3	1.80	2.34	5.89	139.42	10.5
			5	0.83	1.03	2.49	156.75	12.7
		5	3	0.62	2.16	2.86	179.22	12.6
			5	0.63	2.03	3.85	198.48	14.3
200	30	3	3	0.72	2.02	5.47	215.73	17.5
			5	0.37	1.79	3.11	271.85	22.4
		5	3	0.48	1.01	3.83	285.16	23.3
			5	0.71	2.05	4.75	325.39	25.7

CONCLUSION

A multitype capacitated concentrator location problem with a choice of various terminal coverages can be formulated as an integer linear programming problem. The cross decomposition method can be applied to solve this problem because both the primal and dual subproblems are relatively easy to solve, and both subproblems quickly converge. Computational results clearly support the notion that the proposed decomposition-based algorithm is a very effective method of finding both an optimal and a near-optimal solution. It outperforms, in the execution time, the (embedded) Benders decomposition algorithm by very large margins. These results occur because the cross decomposition algorithm maintains a special structure in both the primal and dual subproblems and exploits these special structures. The algorithm solves problems of practical sizes in acceptable times. Therefore, the implementation of this algorithm provides flexible means to model concentrator location problems in computer communication networks.

References

[1] Ahuja, V. Design and Analysis of Computer Communication Networks. McGraw-Hill: New York, 1982.

[2] Bahl, L. R. & Tang, D. T. "Optimization of Concentrator Locations in Teleprocessing Networks". Proceedings: Symposium on Computer Communications and Teletraffic, 1972, 355-362.

[3] Balachandran, V. "An Integer Generalized Transportation Model for Optimal Job Assignment in Computer Networks". Operations Research, 1976, 24, 742-759.

[4] Boorstyn, R. R. & Frank, H. "Large-scale Network Topological Optimization". IEEE Transactions on Communication, 1977, 25, 29-46.

[5] De Maio, A., & Roveda, C. "An All Zero-one Algorithm For a Certain Class of Transportation Properties". Operations Research, 1971, 19, 1406-1418.

[6] Dysart, H. G. & Georganas, N. D. "NEWCLUST: An Algorithm For the Topological Design of Two-level, Multidrop Teleprocessing Networks". IEEE Transactions on Communication, 1978, COM-26, 55-61.

[7] Elias, D. & Ferguson, M. J. "Topological Design of Multipoint Teleprocessing Networks". IEEE Transactions on Communication, 1974, COM-22, 1753-1762.

[8] Erlenkotter, D. "A Dual-Based Procedure for Uncapacitated Facility Location". Operations Research, 1978, 26, 992-1009.

[9] Fisher, M. L. The Lagrangean Relaxation Method For Solving Integer Programming Problems. Management Science, 1981, 27, 1-18.

[10] Frank, H. & Frisch, I. "Optimal Design of Centralized Computer Networks". Networks, 1971, 1, 43-57.

[11] Garey, M. R. and Johnson, D. S. Computers and Intractability: A Guide to the Theory of Np-Completeness. Freeman: San Francisco, 1979.

[12] Gavish, B. "Topological Design of Centralized Computer Networks - Formulations and Algorithms". Networks, 1982, 12, 355-377.

[13] Gavish, B. "Models for Configuring Large Scale Distributed Computing Systems". AT&T Technical Journal, 1985, 64, 491-532.

[14] Greenburg, D. A. "A New Approach For the Optimal Placement of Concentrators in a Remote Terminal Communications Network". Proceedings of National Telecommunications Conference, Atlanta, GA: 1973, 37D1-37D7.

[15] Hoang, H. H. "Topological Optimization of Networks: A Nonlinear Mixed Integer Model Employing Generalized Benders Decomposition". IEEE Transactions on Automatic Control, 1982, AC-27, 164-169.

[16] Holmberg, K. & Jornsten, K. O. "Cross Decomposition Applied to the Stochastic Transportation Problem". European Journal of Operational Research, 1984, 17, 361-368.

[17] Kershenbaum, A. & Boorstyn, R. R. "Centralized Teleprocessing Network Design". Proceedings of National Telecommunications Conference, New Orleans: 1975, 27.11-27.14.

[18] Kershenbaum, A. & Chou, W. "A Unified Algorithm For Designing Multidrop Teleprocessing Networks". IEEE Transactions on Communication, 1974, COM-22, 1762-1772.

[19] Lasdon, L. S. Optimization Theory for Large Systems. Macmillan Co.: New York, 1970.

[20] Lee, C. Y., "An Optimal Algorithm for Multiproduct Capacitated Facility Location Problem with a Choice of Facility Type". Computers & Operations Research, 1991, 18, 167-182.

[21] Lee, C. Y. "A Cross Decomposition Algorithm for a Multiproduct-Multitype Facility Location Problem". To appear in Computers & Operations Research.

[22] Lee, C. Y. "A Heuristic Approach for a Stochastic Transportation-Location Problem with Cross Decomposition". To appear in International Journal of Information and Management Sciences.

[23] Lee, C. Y. "Application of Cross Decomposition Algorithm to a Location and Allocation Problem in Distributed Computer Network Systems". Working Paper No.3, Pittsburg State University, 1992.

[24] Lee, C. Y. "A Two-Staged Distribution Network Problem with Various Types of Distribution Centers". Working Paper No.4, Pittsburg State University, 1992.

[25] Love, R. F., Morris, J. G., & Wesolowsky, G. O. Facility location: Models and Methods. North-Holland, New York: 1988.

[26] Love, R. F., Truscott, W. G. & Walker, J. H. "Terminal Location Problem: A Case Study Supporting the Status Quo". Journal of Operational Research Society, 1985, 36, 131-136.

[27] McGregor, P. & Shen, D. "Network Design: An Algorithm For Access Facility Location Problem". IEEE Transactions on Communication, 1977, COM-25, 61-73.

[28] Mirzaian, A. "Lagrangian Relaxation For the Star-Star Concentrator Location Problem: Approximation Algorithms and Bounds". Networks, 1985, 15, 1-20.

[29] Neebe, A. W. & Rao, M. R. "An Algorithm For the Fixed-Charge Assigning Users to Sources Problem". Journal of Operational Research Society, 1983, 34, 1107-1113.

[30] Nemhauser, G. L. & Wolsey, L. A. Integer and Combinational Optimization. Wiley-Interscience: New York, 1988.

[31] Pirkul, H. "Efficient Algorithms For the Capacitated Concentrator Location Problem". Computers and Operations Research, 1987, 14(3), 197-208.

[32] Sankaran, J. K. "A Cross Decomposition Algorithm For Routing Passenger Vehicles". MS Thesis, The University of Iowa, 1986.

[33] Schneider, G. M. & Zastrow, M. N. "An Algorithm For the Design of Multilevel Concentrator Networks". Computer Networks, 1982, 6, 1-11.

[34] Schrage, L. Linear, Integer and Quadratic Programming with LINDO, Scientific Press, 1986.

[35] Srinivasan, V., & Thompson, G. L. "An Algorithm For Assigning Uses to Sources in a Special Class of Transportation Problem". Operations Research, 1973, 21, 284-295.

[36] Tang, D. T., Woo, L. S., & Bahl, L. R. "Optimization of Teleprocessing Networks With Concentrators and Multiconnected Terminals". IBM Research Report RC5149, 1974.

[37] Van Roy, T. J. "Cross Decomposition for Mixed Integer Programming". Mathematical Programming, 1983, 25, 46-63.

[38] Van Roy, T. J. "A Cross Decomposition Algorithm for Capacitated Facility Location". Operations Research, 1986, 34, 145-163.

[39] White, L. J. "Optimum Concentrator Locations in Telecommunication Design". Association of Computing Machinery Conference Proceedings, 1972, 534-541.

[40] Woo, L. S. & Tang, D. T. "Optimization of Teleprocessing Networks With Concentrators". Proceedings of National Telecommunications Conference, Atlanta, GA: 1973, 37C1-37C5.

[41] Yoo, C. and Tcha, D. "A Cross Decomposition Procedure for the Facility Location Problem with a Choice of Facility Type". Computers & Industrial Engineering 10, 283-290 (1986).